4EU+ Student Mini-grants 2022 - Charles university

The mathematics of the Whitechapel

A board game analysis from the mathematical point of view.

Research diary

M. Cattazzo, E. Cortinovis, J. Hercík, Ł. Kamiński, P. Szlufik, L. Wintrová

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Abstract

Nothing better than a "hands on" project can help developing flexible learning paths: having to merge different backgrounds to form a consistent team working on a topic which is previously unknown to everybody implies the developement of a fast, shared, aware and precise learning method. If we want to add to the mix some amount of international mobility, a 21st-century smart approach, involving delocalized, task based work becomes fundamental as well.

Likewise, there's no better training environment for a mathematician than a board game: as a pre-simplified, axiomatic reality the player willingly chooses to enter, every mathematical result acquires solidity, holding an immediate impact on the whole gameplay.

Tempted by the succulent learning occasion, at the beginning of September we took part in a project funded by Charles University as a member of 4EU+ university Alliance, that put together all those previously mentioned aspects.
The project also involved research-based education, accessible to undergraduate students, making it a favourable context to make some steps forward to developing some efficient active learning methods for students to enjoy.
In the following pages we collected some highlights of our experience, to share our experience and contributions to the cause.

The project consisted in getting as close as possible to finding a winning strategy for the game Letters from Whitechapel (by Gabriele Mari and Gianluca Santopietro), an Italian-made, 21st-century, non-random, complete information, asymmetrical, two-sides competitive game. It takes place on an unusual board formed by two interconnected graphs, where a team of five policemen hunts down Jack the Ripper.

Chapter 1

Prologue

1.1 How the team assembled

In the summer of 2021, Lucie Wintrová met Łukasz Kamiński in Milan, while participating in the *Bachelor in Mathematics Student Task* project organized by the 4EU+ Alliance and supervised by the University of Milan. Later that year, Marco Cattazzo, Emanuele Cortinovis and Patryk Szlufik took part in the same project under the supervision of Charles University in Prague. There, they all acquainted Lucie who had gone to see the final presentation about the studies they had carried out during the aforementioned project. Therefore, when the opportunity to work on a new project arose thanks to the student mini-grant offered by the Charles University and the 4EU+ Alliance, the soon-to-be team members, eager to seize this opportunity, naturally contacted each other. The last person to join the team, Jakub Hercík, was invited by his friend and colleague Lucie. The idea to cover the topic of an optimal or advantageous game strategy came from Marco who is an avid player of board games. The game he suggested to the group was "Letters from Whitechapel".

1.2 Before coming to Warsaw

After it had been agreed that we were all interested in pursuing this research topic, we laid down the logistics thanks to the several online platforms internet had to offer, so that we could cooperate more easily. The main product of the work, consisting in some LaTeXand matlab files, was stored in an online cloud, and on a daily basis we communicated via group chat in the WhatsApp application. We also held video calls whenever necessary. It was during such a video call we discussed several ways the research could be approached, and consequently decided it would be of a great benefit to meet in person at least for several days. This was in order to allow us efficiently put together all of our ideas, as it was rather difficult to do so via online means only. Subsequently, we filled an application for the grant, which was accepted.
When it comes to the way we worked prior to coming to Warsaw, it was mostly individually or in smaller groups of two or three people, since we all have different interests and fields of study. Therefore each of us firstly tried to approach the problem with the tools they were comfortable with.

We have learned that...

In order to start a research project in mathematics, it is fundamental to have an **aim** or a goal, and an **idea** on how to pursue it. In our case, the aim was both to prove the existence of a winning strategy and to create some practical tools in order to help both sides during the game.

Chapter 2

The first day in Warsaw - Aug 31^{st}, 2022

The Wednesday's program began with a commute from our accommodation to the grounds of the University of Warsaw, our de facto home for the entirety of our stay.

Morning Session

We started with an exchange of the ideas we did not manage to write down into the shared online workspace before our respective departures to Warsaw as well as with discussing the already recorded results in depth. Subsequently, an approximate schedule of our work for the rest of the week was made. However, it would slightly change over time as we were coming up with new ideas or building upon the already existing ones. Some of the main points we set for ourselves included the following:

- To develop effective tools for the planning of the strategy of each side.
- To write programs incorporating some of these tools and their visualization.
- To try to describe the game in the form of axioms in order to prove or disprove the existence of pure winning strategies.

Afternoon Session

After lunch we moved our equipment to another room which contained a larger whiteboard and an interactive screen, so that we could discuss our ideas more efficiently. We also examined together some of the results of our morning work. After this we started working in smaller groups on different branches of the research with periodic breaks to discuss the progress (or the lack of it) together. The focus was mainly on the points mentioned above and also on the prospective necessity (or convenience) of some sort of visualisation.

We have learned that...

It is critical to choose how to appropriately **split** the work, so that the team can complete the task in an efficient and clear way.

We have learned that...

It is very important to find a suitable and flexible working **environment** which allows presenting ideas to others easily, but also for each one to carve out some space to think. Moreover, equipment meant for relaxation and stimulation such as armchair or coffee maker might be quite useful for resting and preparing for subsequent work.

We have learned that...

When one encounters a roadblock it is often wise to assist someone with a different work until the right idea comes up or the fog clears. Not only you help a friend, but you also might encounter something which will help you with your own problem.

Chapter 3

The second day in Warsaw - Sept. 1^{st}, 2022

Morning Session

The second day started with a contemplation about whether (and eventually how) it would be possible to utilize some sort of graph isomorphism to ease the logic axiomatization. We also discussed for a long time the matrix approach, i.e. using adjacency matrices to describe our problem. In the end, we decided to create an interactive computer representation of the game board connected to graph matrix, so that we could use it in our optimisation algorithms. Several possible applications of the matrix approach in the proving the existence or non-existence of pure winning strategy were also inspected. As an interesting note, it was during the morning on the 1st of September that the idea to write this diary was born; and, subsequently, the work on it began.

Afternoon Session

In the afternoon it became clear that even though the computational approach, the linear algebraic approach and the abstract algebraic approach had nontrivial intersection, in some aspects they would inevitably diverge. When it comes to the intersection of matrices and programming, we originally planned to use only the white graph and its adjacency matrix, however gradually it became clear that the graph of black policemen's crossings would be needed as well. More precisely, we had to construct and calculate with the combination of the two matrices.
Because of the fact that we did not manage to find any practical software which would allow us to transform the game board graph into its representation in a computer, we had to do it manually, as developing such tool could have easily been a project of its own. Nonetheless, the most notable thing of the afternoon session might be the start of the creation of some of the visualisation tools.

We have learned that...

Finding some **literature**, and following the references in search of some useful **instrument/concept/idea** might be an interesting strategy to tackle a problem. However, a certain amount of caution must be exerted with this method, as in the long run it might become quite time-consuming.

We have learned that...

Visualisations allow us to represent and enjoy the fruits/results of the work, but more importantly to make up our mind in a synthetic and qualitative way in order to choose what to do next. Nonetheless, as with the literature, one has to carefully consider whether the time investment is going to pay for itself.

Chapter 4

The third day in Warsaw - Sept. 2^{nd}, 2022

Morning Session

On the night between the second and the third day Emanuele arrived from Italy after tackling unavoidable matters linked to his study. Even though he was continuously following the work we had been doing up to that point, we decided to summarize the results from the previous two days for him once again. We also included him into the workflow and discussed with him where his help would be the most useful.
As for the mathematical work, we were trying to utilize shortest path algorithms in both theoretical and practical fashion. However, since the game board graph is a bit "messy", it is sometimes difficult to individuate which vertices share an edge. Because of this, we discovered we had made some small mistakes in setting up its "electronic form" the previous day, and we had to correct them. Aside from this however, the computational approach utilizing the matrices and linear algebra in general seemed to work quite well. Nonetheless, there were still some remaining challenges stemming from the rules of the game.

Afternoon Session

In the afternoon we mostly continued with the work from the morning. The addition of Emanuele brought forth some new ideas we tried to incorporate into what we already had. One new goal we set for ourselves in the afternoon was to uniform the notation, which, despite our best efforts, suffered a bit from the fact that each of us, coming from a different background, had been inclined to use it in its own way. We have also encountered a series of various minor obstacles.

We have learned that...

It is hard to evaluate the real complexity of the problem in advance. The challenge could be harder than it appeared at the beginning, when the problem was chosen. The effectiveness of the selected strategy might be lower than expected, or the results, upon further inspection, might not be the forecast ones. This could cause some amount of time loss, which might be significant in case the resources deployed for the research are limited. Especially, choosing a theorem to prove in a mathematical research is a real bet, because rigorous proofs are often rather tricky things to construct.

We have learned that...

It is often helpful to pause the workflow to **discuss** the results from individual branches of the research all together. Some of the results might be invaluable to some other paths.

Chapter 5

The fourth day in Warsaw - Sept. 3^{rd}, 2022

Morning Session

As our stay was coming to an end, we held a long joint morning meeting. The goal was to summarize what we had came up with up to that point and to set out what needs to be done in order to compile our work and finish the project. We also took a group photo with the game and our main visualisations of it.

Afternoon Session

During the afternoon we mostly continued writing down some of the ideas we came up with during the week and adding finishing touches on the visualisations as well as other coding parts (mainly on the program which determines where the hideout is based on the investigations conducted by the policemen).

Evening Session

The fourth day differed from the previous ones by having an evening session. However, we didn't exactly work per se, we were rather "playtesting" some of the tools we had developed. We all gathered at our accommodation and played the game together, although with one noteworthy modification. During the play, all investigations and their results were input into our "hideout search algorithm" which after a few rounds correctly determined where the hideout really was, even long before the policemen had any intuition of the precise location.
Sadly, this was the last piece of work we have done together in Warsaw. The very next morning the two Czech members of the team departed back home and the Italians followed soon after. Nonetheless, there was still work to be done. We still had to put together our results, for example, and also to finish this diary. We have done all of this in the following weeks.

We have learned that...

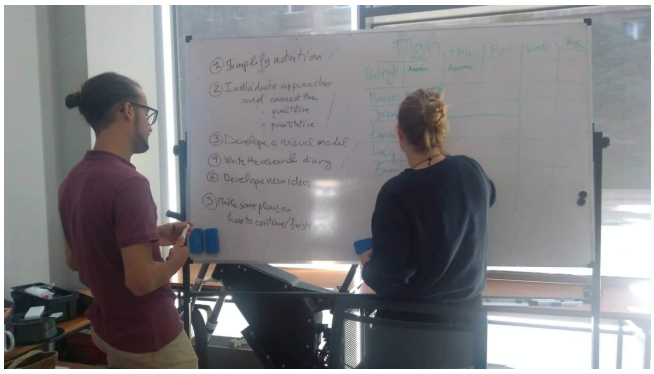
Throughout the week we spent together in Warsaw as well as during the online collaboration we have learnt that **the challenges of working in a team require some practice.** For instance, it is sometimes difficult to decide whether is it better to work together or to split and pursue separate paths at a given moment.

We have learned that...

A great obstacle we encountered before accepting the challenge of an undergraduate/graduate research is **the fear of not coming up with ideas**, which fear we soon discovered to be completely unfounded: everyone sparred corageously by bringing to the campaign whichever little piece of knowledge he possessed to begin with, and investigating whichever smallest intuition arising from the group discussion. During the course of our studies, we unavoidably and noticeably sharpen our intuition and broaden our experience baggage, which turns out to be extremely useful when it comes up to open-sea, unsupervised research. Whatever is left to do is to acquire awareness of this process.

Chapter 6

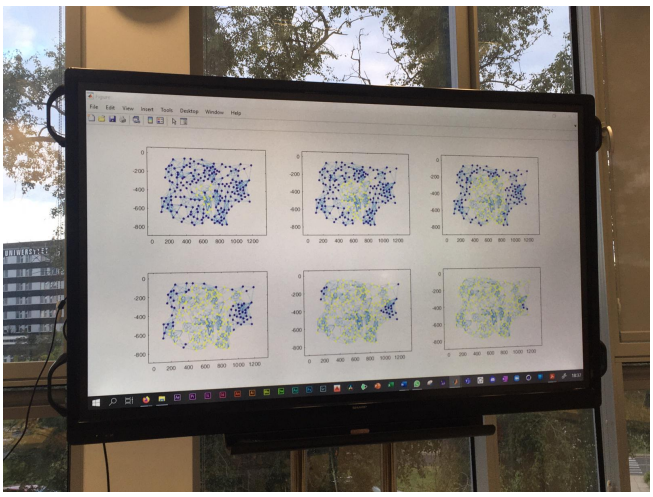
Photos



The making of our schedule

Programming

Brainstorming



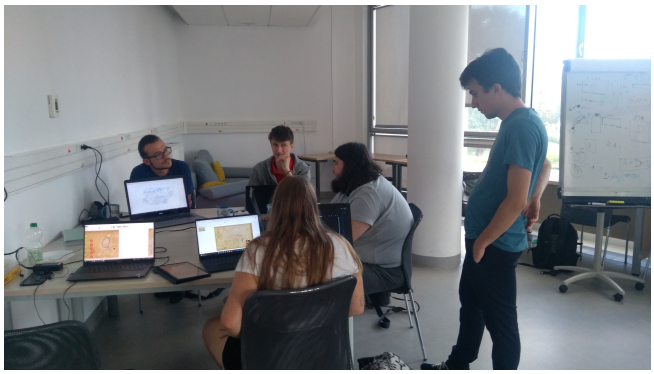
One of the developed visualizations

Another visualization

And its development

A much needed coffee

Contemplation of the matrix approach

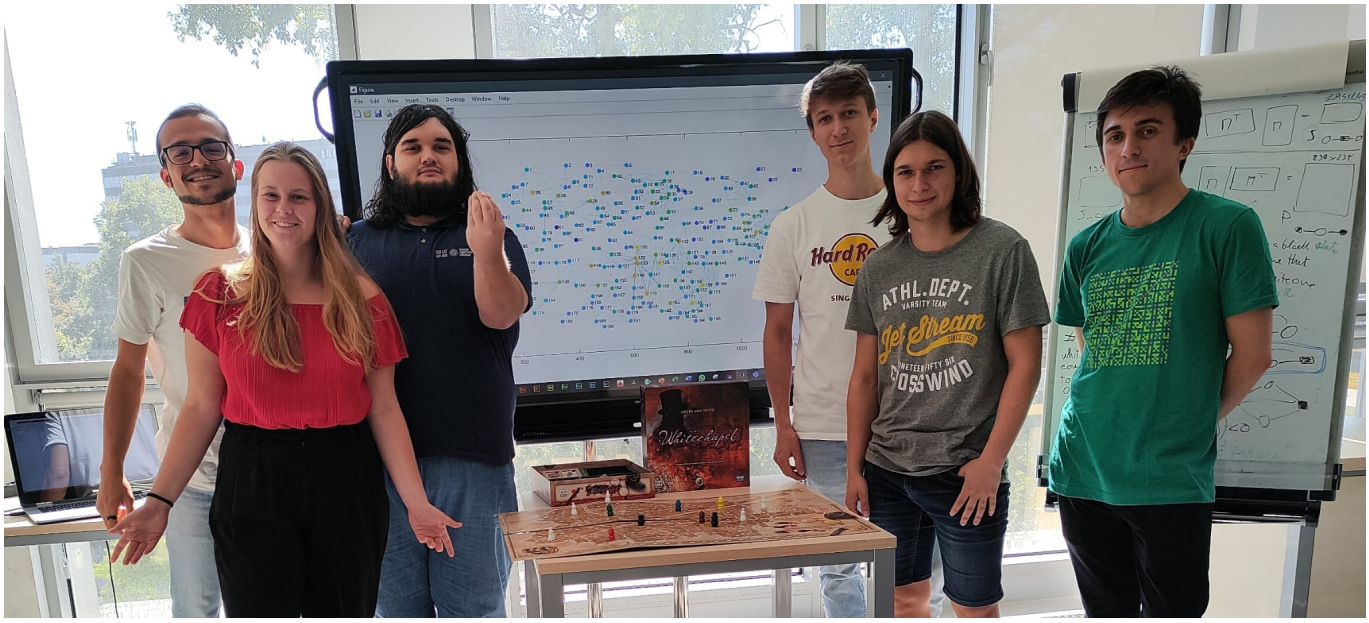


Yet another brainstorming

Writing down the ideas

The manifestation of ideas

The game



From left: M. Cattazzo, L. Wintrová, P. Szlufik, E. Cortinovis, J. Hercík, Ł. Kamiński

Chapter 7

Epilogue

We would like to express our deepest gratitude to the 4EU+ alliance and Charles University for funding this research, thus enabling us to build new relationships, to experience working together on a project of such scale and to contribute with our results to the collective knowledge of humanity.